Mathematics
Higher level
Paper 3 – sets, relations and groups

Wednesday 18 May 2016 (morning)

1 hour

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[60 marks]**.

3 pages

2216 – 7209

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 19]

The following Cayley table for the binary operation multiplication modulo 9, denoted by $*$, is defined on the set $S = \{1, 2, 4, 5, 7, 8\}$.

$*$	1	2	4	5	7	8
1	1	2	4	5	7	8
2	2	4	8	1	5	7
4	4	8				
5	5	1				
7	7	5				
8	8	7				

(a) Copy and complete the table. [3]

(b) Show that $\{S, *\}$ is an Abelian group. [5]

(c) Determine the orders of all the elements of $\{S, *\}$. [3]

(d) (i) Find the two proper subgroups of $\{S, *\}$.

(ii) Find the coset of each of these subgroups with respect to the element 5. [4]

(e) Solve the equation $2 * x * 4 * x * 4 = 2$. [4]

2. [Maximum mark: 12]

The relation R is defined on $\mathbb{Z}^+$ such that aRb if and only if $b^n - a^n \equiv 0 (\text{mod } p)$ where n, p are fixed positive integers greater than 1.

(a) Show that R is an equivalence relation. [7]

(b) Given that $n = 2$ and $p = 7$, determine the first four members of each of the four equivalence classes of R. [5]

3. [Maximum mark: 7]

The group $\{G, *\}$ is Abelian and the bijection $f: G \to G$ is defined by $f(x) = x^{-1}$, $x \in G$. Show that f is an isomorphism.

4. [Maximum mark: 13]

The function f is defined by $f: \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+ \times \mathbb{R}^+$ where $f(x, y) = \left(\sqrt{xy}, \frac{x}{y}\right)$.

(a) Prove that f is an injection. [5]

(b) (i) Prove that f is a surjection.

(ii) Hence, or otherwise, write down the inverse function f^{-1}. [8]

5. [Maximum mark: 9]

The group $\{G, *\}$ is defined on the set G with binary operation $*$. H is a subset of G defined by $H = \{x : x \in G, a * x * a^{-1} = x$ for all $a \in G\}$. Prove that $\{H, *\}$ is a subgroup of $\{G, *\}$.